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TECHNICAL NOTE

DISTRIBUTION OF RESPONSE RATIOS IN CONCURRENT VARIABLE-INTERVAL PERFORMANCE[1]

A recent editorial (Zeiler, 1977) emphasized the importance of describing the variability of data within conditions, and not simply measures of central tendency. The present Note describes the variability of concurrent schedule response ratios within experimental conditions.

Concurrent schedules were arranged according to the two-key method (Herrnstein, 1961) on keys transilluminated white. Variable-interval (VI) schedules of reinforcement were composed of 12 intervals in random order, generated from an arithmetic progression, a, a + d, a + 2d, *etc.*, where a = d/2. The schedules were arranged nonindependently; that is, when reinforcement was arranged on one key, both VI schedules stopped (Stubbs and Pliskoff, 1969). A changeover delay of 3 sec was used (Herrnstein, 1961). The schedules of reinforcement were changed when all six animals had met a stability criterion five times, not necessarily consecutively. The criterion required that the median relative number of responses over five sessions was within 0.05 of the median of the previous five sessions. Numbers of responses on each key of concurrent schedules were recorded every time pigeons changed from responding on the left key to responding on the right key on the last five days of experimental conditions.

Table 1 shows the sequence of experimental conditions, the numbers of responses and reinforcements obtained on the right and left keys, the numbers of training sessions, and the numbers of response samples taken from each condition.

Each sample of response numbers was converted to a log response ratio and a frequency distribution of these ratios in 0.1 log units was derived for each condition. This distribution was then converted to a cumulative frequency distribution for analysis. The simplest way of demonstrating the normality of a distribution is to convert the cumulative frequencies into z-scores. If z-scores fall on straight lines, the distributions are normal. The slope of the straight line is a measure of the standard deviation of the distribution.

Figure 1 shows the z-scores from 1 to 99% of the cumulative frequency distribution for each animal. Straight lines were fitted to the data of Figure 1 using the method of least squares, giving the results shown in Table 2. The variance accounted for by the straight lines was not less than 94% for any of the 30 fitted lines, showing that the distributions of response ratios closely approximated normal distributions. Both the slopes and intercepts of the fitted lines were tested for trend across changing relative reinforcement rates using a nonparametric trend test (Ferguson, 1966). No significant trend in the slopes of the fitted lines was found, indicating that the standard deviations of the distributions did not change between conditions. The intercepts of the fitted lines did decrease as relative reinforcement decreased ($p < 0.01$), showing that response ratios follow reinforcement ratios in concurrent schedule performance (Herrnstein, 1961).

Matching or undermatching is measured from the slopes of least-squares lines fitted between log response ratios and log obtained reinforcement-rate ratios (Baum, 1974). For the least-squares method to provide unbiased estimates of slopes and intercepts, two requirements must be met. These are that the values of the dependent variable are normally distributed, and that the variance of these distributions is the same for each value of the independent variable. Figure 1 shows that both of these requirements are met for log response ratios, so the method of least squares is appropriate for these log response ratio and log reinforcement ratio data. Note that if relative response rates ($P_1/P_1 + P_2$) had been used instead of the ratio of response rates (P_1/P_2), then the ceiling and floor of the relative measures would have distorted the shapes of the distributions of the dependent variable. The present results show that the least-squares method of curve fitting is appropriate for log response ratio and log reinforcement ratio data (and also for the z-transform of relative measures, Bush, 1963), but that it cannot give unbiased estimates when untransformed relative measures are used.

The slopes of the least-squares lines fitted between log response ratio and log reinforcement ratios for the data summed over the last five sessions are shown in Table 3. Five of the six subjects showed undermatching of response ratios to the reinforcement ratios (that is, slopes in Table 3 of less than 1.0). Lobb and Davison (1976) and Myers and Myers (1977) have reviewed evidence suggesting that undermatching is a common finding in concurrent VI VI schedules. A plausible explanation of undermatching might be that response distributions at extreme reinforcement ratios were nonnormal, with a tail towards indifference. The data presented here show that undermatching in these sub-

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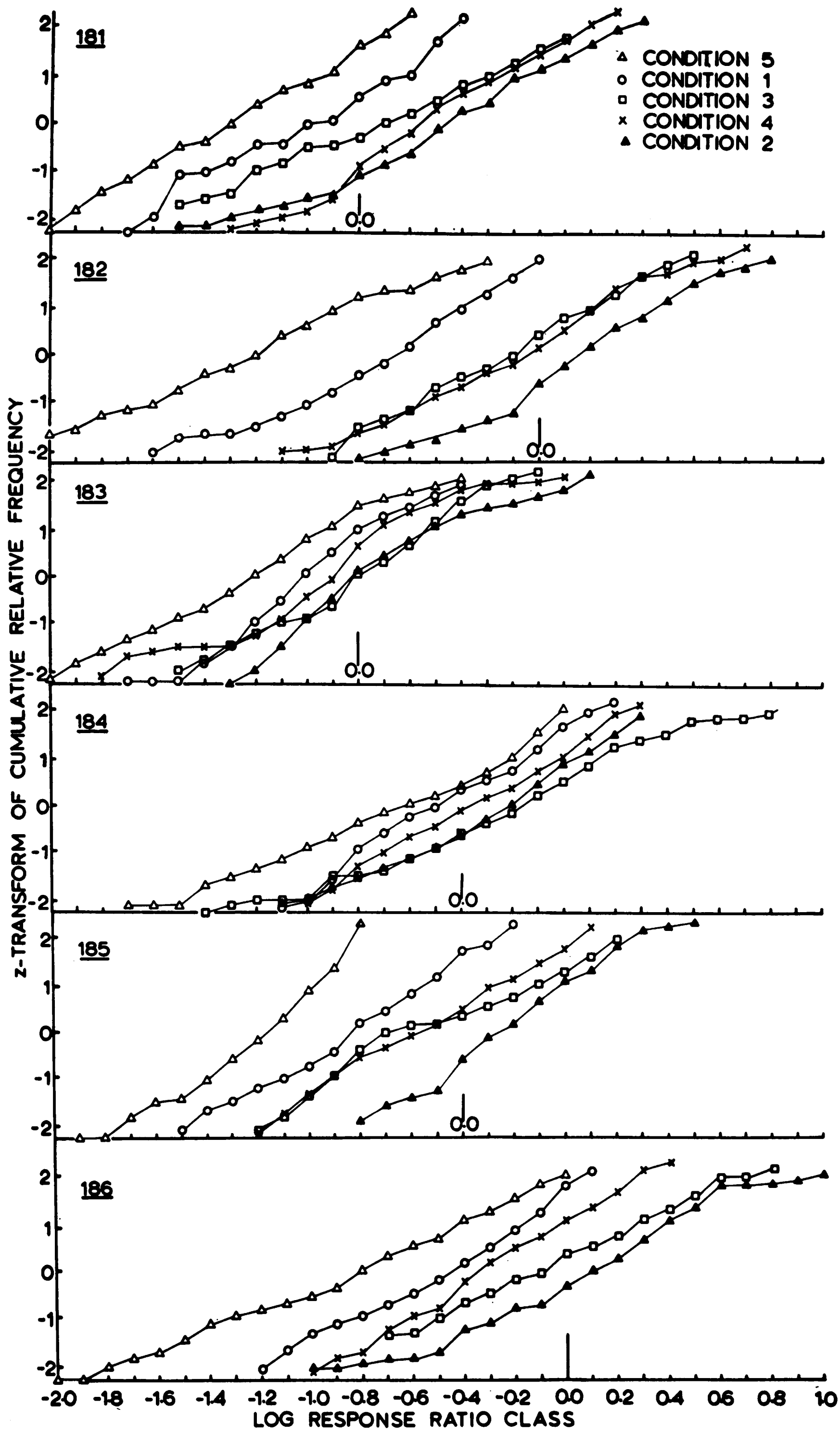

Fig. 1. The z-transform of the cumulative relative frequency of occurrence of log P_1/P_2 samples in successive 0.1 class intervals of log (P_1/P_2). Note that the origin of the abscissa is in a different location for each bird.

Table 1

Sequence of experimental conditions, number of sessions training, and numbers of responses and reinforcements on the right and left keys. The data are sessional averages taken over the last five sessions of each condition. The total number of response-ratio samples analyzed is also shown.

VI Schedules (seconds)			*Responses*		*Reinforcements*			
Right	*Left*	*Bird*	*Right*	*Left*	*Right*	*Left*	*Sessions*	*Samples*
60	30	181	403	507	17.2	32.8	23	301
		182	447	1403	17.0	33.0		177
		183	828	1080	17.0	33.0		321
		184	1207	1254	17.0	33.0		315
		185	384	839	16.2	33.8		189
		186	490	1194	17.4	32.6		236
30	120	181	769	287	39.6	10.4	26	219
		182	1273	611	40.6	9.4		164
		183	1319	909	40.4	9.6		372
		184	1835	803	40.8	9.2		275
		185	951	550	40.6	9.4		302
		186	663	430	39.8	10.2		165
30	30	181	543	298	25	25	23	172
		182	574	658	25	25		156
		183	814	639	25	25		240*
		184	1236	594	25	25		194
		185	540	455	25	25		127
		186	396	407	25	25		138
30	60	181	685	306	33.4	16.6	19	230
		182	752	791	32.4	17.6		258
		183	1087	1055	33.2	16.8		395
		184	1571	1155	32.4	17.6		248
		185	773	720	33.0	17.0		228
		186	510	882	33.2	16.8		199
120	30	181	445	1005	10.0	40.0	27	303
		182	143	1874	10.4	39.6		75
		183	841	2063	9.2	40.8		346
		184	1286	2005	10.0	40.0		269
		185	256	1165	10.2	39.8		143
		186	298	2017	9.2	40.8		100

*Sample is for four sessions only.

jects did not result from nonnormal log response-ratio distributions.

The analysis showed that the distributions of response ratios in these concurrent schedules were approximately normal for each reinforcement ratio. The shapes and standard deviations of the response-ratio distributions did not change when reinforcement ratios changed. It remains to be shown that the present results apply to other subjects, schedules and scheduling procedures (for example, independent concurrent VI VI schedules). If this is shown, measures of variability in each condition may be unnecessary, a single measure across conditions sufficing. While a Friedman 2-way analysis of variance (Siegel, 1956) revealed no significant between-subject differences in standard deviations of the distributions (the slopes in Table 2), caution should be exercised in fitting data obtained from more than one subject and in fitting grouped data. Even more hazardous, in terms of biasing the results, would be a fit to data obtained in different experiments.

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Table 2

Slopes and intercepts relating the z-transform of the cumulated relative frequencies of emission of a class log response-ratio measures as a function of that class log response-ratio measure. The conditions are shown in the natural ordering of increasing relative reinforcement on key 2. V refers to the percentage of variance accounted for by the line fitted by the method of least squares.

Bird	*Condition*	*Slope*	*Intercept*	*V*
181	5	3.17	1.59	100
	1	1.13	0.66	97
	3	2.40	−0.17	99
	4	3.41	−0.87	99
	2	2.71	−0.85	97
182	5	2.39	2.62	98
	1	2.82	1.69	96
	3	3.00	0.34	99
	4	2.98	−0.54	98
	2	2.72	0.23	98
183	5	2.88	1.22	99
	1	3.57	0.64	97
	3	3.34	0.08	99
	4	2.76	0.41	94
	2	3.23	−0.28	95
184	5	2.46	0.62	98
	1	3.46	0.25	99
	3	2.26	−0.46	98
	4	3.25	−0.14	100
	2	3.02	−0.41	98
185	5	4.12	3.55	95
	1	3.15	1.43	97
	3	2.70	0.33	97
	4	3.25	0.52	100
	2	3.61	−0.58	99
186	5	2.26	1.93	99
	1	3.13	1.59	99
	3	2.60	0.32	100
	4	3.37	1.07	100
	2	2.47	−0.14	97

Table 3

The relation between the logarithms of the response and reinforcement ratios, obtained by the method of least squares over all five conditions of the experiment. SD refers to standard deviation, and V to the percentage of variance accounted for by the fitted line.

Bird	Slope	SD	Intercept	SD	V
181	0.68	0.11	0.12	0.05	93
182	1.12	0.20	−0.28	0.09	91
183	0.40	0.11	−0.04	0.05	81
184	0.41	0.14	0.12	0.06	75
185	0.70	0.15	−0.13	0.07	88
186	0.73	0.20	−0.25	0.09	81

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